



Update Migration Study Twenthe-Rijn – Phase I

Report Part 2 - The Bekendam-Jakob-Pinkse (BJP) subsidence prognosis algorithm for migrating salt caverns in a bedded stratigraphy

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1 Executive summary and Report Structure

The first part of the overall reporting for the Update Migration Study (UMS), delivered in December 2023 [1], describes the fundamental principles used during the reconstruction of leaching and migration development of the 10 caverns included in the study. The aim of this report was to explain the fundamental principles related to the reconstruction of cavern development, slurry filling, migration, as well as the consequential link to subsidence, to support the reader in the understanding of the further parts of the overall reporting.

This second part of the overall reporting for the UMS describes the Bekendam-Jakob-Pinkse subsidence prognosis algorithm for migrating salt caverns in a bedded stratigraphy. This algorithm has been developed based on the formerly used “Bekendam-Method” for subsidence prognosis and several insights from the former step in the UMS, i.e. the reconstruction of cavern development and migration for 10 well documented cases of cavern migration in the Hengelo brinefield. Intermediate reporting on these reconstructions has been delivered in the form of a presentation slide-deck [2]. A detailed description of the reconstructions, and the consequent validation of the algorithm, will be included in Report 3, titled “Validation of the input parameters by the assessment of migration behaviour of migrated caverns and determination of the likelihood of migration”, which is being drafted at the time of writing this document.

The Bekendam-Jakob-Pinkse (BJP) prognosis algorithm has been successfully validated using actual subsidence data for the studied caverns during Phase 1 of the UMS. The cavern-specific input parameters have been selected based on a unified workflow considering the cavern roof position in the stratigraphy, the actual cavern volume, the potential migration column radius, and the likely cavern filling grade by bulked overburden. The selected input parameters transform an irregularly shaped cavern into a corresponding basic cavern shape (cylinder- and/or cone-shaped) which can be used as input for the BJP algorithm. Intermediate reporting on the selection of input parameters and validations has been delivered in the form of two presentation slide-decks [3], [4]. A detailed description of these validations will be included in Report 3. Conclusions with regards to the applicability and conservatism of the BJP-Algorithm, as well as ample comparisons with the formerly used Bekendam method will also be included in this report.

The cavern specific selection of input parameters following the unified workflow, as well as the results from the prognoses will be included in Report 4, titled “Subsidence predictions for potential future migration and recommendations on mitigative action by means of backfilling which is expected to be delivered in conjunction with Report 3.



The BJP prognosis algorithm has been implemented in a Python¹ script to enable a fast turnaround of calculations and output generation by automating several otherwise time-consuming steps. The BJP-method includes the following new features to the subsidence prognosis capacity compared to the formerly used (Bekendam) method:

1. Three (3) basic cavern shapes: cylinder, cone and cylinder + cone.
2. Three (3) consequential migration column shapes: cylinder, cone, cylinder + cone.
3. A mechanism describing the consolidation of backfill based on brine purification residues under its own weight (i.e. self-weight-consolidation (SWC)) and under the load of bulked overburden after cavern migration has been initiated.
4. A migration column radius reduction mechanism for cone shaped caverns based on the hindrance of bulking near the outer cavern contour of a cone-shaped cavern.
5. Deflection of the (residual) cavern roof beam (i.e. horizontal pillar) near the top rock (i.e. bottom Tertiary) dependent on the migration column radius at the critical depth as an intermediate between subsidence by migration void deformation and sinkhole/trough formation.
6. Superposition of multiple subsidence sources (i.e. migration columns) for overlapping subsidence areas to achieve an overall subsidence prognosis for an area where multiple caverns for which migration cannot be ruled-out are present.

The BJP-method comprises a pre-processing, a calculation and a post-processing step to achieve a subsidence prognosis for a migrating cavern. Pre-processing entails the selection of cavern specific input parameters such as cavern diameter, height, volume, etc. The script calculates subsidence, tilt, strain, curvature and relative rotation (for a pre-determined building-length) and outputs both a summary table and a series of geo-referenced contours for each of the mentioned output parameters. Both contours per cavern/migration column individually as well as superpositioned contours for overlapping subsidence areas can be generated using the BJP script. The post-processing step achieves a legible reporting product from which the results of the prognosis can be easily read and understood.

The detailed description of the BJP prognosis algorithm included in the present report is divided into two chapters. Chapter 2 covers the basic input parameters and preliminary calculations used to feed the subsidence prognosis algorithm, which is discussed in Chapter 3. The report is concluded with tables of references, figures and tables.

1 Python is a well-known, high-level, general-purpose programming language.



2 Basics, parameters and preliminary calculations

2.1 Basic denotations and indexes

Table 1 gives an overview of the most important denotations and indexes for the BJP subsidence prognosis algorithm, which is based on the Bekendam subsidence prognosis method [5], [6]. Indexes are frequently used several times for one denotation. In these cases, the indexes are separated by commas (e.g. $d_{Tcav,act}$ means depth of the actual cavern top in meters below ground level).

Table 1 Basic denotations and indexes in the BJP algorithm

symbol	unit	description
denotations		
d	mbGL	Depth (always given in meters below ground level, i.e. increasing with depth)
H	m	Height (vertical distance)
R	m	Radius
V	m ³	Volume
η	m	Surface subsidence
f_{consol}	-	Consolidation factor for backfill
indexes		
X_{TR}	-	Index for top of overburden rock (top rock)
X_{cav}	-	Index for cavern
X_{Tcav}	-	Index for cavern top
X_{Bcav}	-	Index for cavern bottom
X_{cyl}	-	Index for cylindrical part of the cavern
X_{cone}	-	Index for conical part of the cavern
X_{ini}	-	Index for initial state of the cavern
X_{act}	-	Index for actual state or starting state for calculation algorithm
X_{mig}	-	Index for migration volume
X_{crit}	-	Index for critical depth (below top rock)
X_Y	-	Index for stage Y subsidence for Y in {2, 3, 4, 5}
X_{hmb}	-	Index for half-height of the migration body
X_{final}	-	Index for final stage of cavern migration
$X_{(no\ backfill)}$	-	Index indicating the absence of backfill
X_{solids}	-	Index for unconsolidated backfill solids

2.2 Input parameters

Most cavern shapes can be idealized by a cylinder, a cone or a mixture of both (see Figure 1).

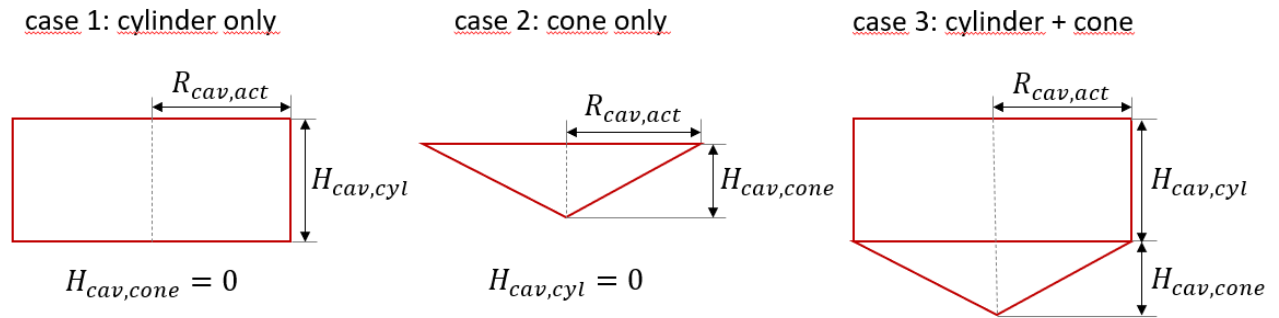


Figure 1 Basic cavern shapes with denotations

Along with the cavern radius $R_{cav,act}$, $H_{cav,cyl}$, $H_{cav,cone}$ and $d_{Bcav,act}$ are the input parameters for the cavern dimensions and location as shown in Figure 1. The top cavern depth is derived by subtracting the cone height and the cylinder height from the depth of the cavern bottom (denotations in Table 1).

$$d_{Tcav,act} = d_{Bcav,act} - H_{cav,cyl} - H_{cav,cone}$$

In addition, the depth of the initial cavern $d_{Bcav,ini}$ is needed as input parameter in BJP algorithm. For geological conditions the top of the overburden rock d_{TR} is the basic input parameter.

Other input parameters are listed in the table below. These parameters are fixed.

Table 2 Fixed parameters (independent from cavern geometry and geological setting)

Parameter	symbol	value	unit	source
Mean unit weight of water	γ_w	10000	Pa/m, N/m ³	(-)
Mean unit weight of soil overburden	γ_s	20000	Pa/m, N/m ³	[5], [6]
Bulking factor	B	1.2	-	[2], [3]
Max. consolidation coefficient*	C_m	3.5 ... 7.0e-9	m ² /N	[5], [6]
Angle of draw (degrees)	γ	45	°	[5], [6]
Litwinsky coefficient	n	4	-	[5], [6]
Strain coefficient	C_s	0,1	-	[5], [6]
Cohesion of top rock	c	2.8 ... 4.0	MPa	[7], [8]
Friction angle of top rock	φ	21.3 ... 26.0	°	[7], [8]
Elastic modulus of top rock**	E	550	MPa	[8]

* Consolidation of bulked overburden by loading with Tertiary soils after the formation of a sinkhole or trough.

** Conservatively reduced by factor 10 (5500/10 = 550 MPa)

2.3 Input parameters for slurry-backfilled caverns

If the cavern was backfilled with a slurry comprising saturated brine and solid brine-purification residues (mainly calcium-carbonate and gypsum), initial cavern height must be reduced by equivalent effective backfill height. The height reduction is based on the volume of in situ backfill as determined by comparison of sonar images/volume reconstructions before and after backfilling.

When before and after sonar images are not available, the slurry volume is derived from historical production figures. A factor of five (5) is used as the Slurry over Backfill volume ratio, meaning that for every five (5) cubic meters of slurry pumped to a cavern, one (1) cubic meter remains as unconsolidated ("fresh") backfill.

"Fresh" backfill in a cavern will initially only consolidate under its own weight. Analysis of (historic and recent) sonar measurements and the preparation of volume balances for the slurry fill have shown that 15% self-weight consolidation (SWC) is an appropriate value.

For the BJP algorithm a value of 15 % self-weight consolidation is used during the selection of input parameters to be used in the prognosis.

The effective backfill height after SWC, i.e. capable of bearing any load, is the total backfill height as determined by sonar images or via the slurry volume minus the thickness of the unconsolidated in-situ backfill at the top of the backfill body in the cavern. From field experiments in backfilled caverns in the Hengelo brinefield it is found that the thickness of this unconsolidated, non-load-bearing layer is 1 - 2 m [9].

For the BJP algorithm a conservative value of $H_{cav,corr} = 2$ m has been selected and the application of this thickness results in an updated cavern height $H_{cav,cyl,backfill}$, which is used in the prognosis.

The just described reduction of actual cavern height $H_{cav,act}$ by firstly SWC of fresh backfill and secondly correction of cavern height by subtraction of the 2 m layer of non-load-bearing backfill ($H_{cav,corr}$), is illustrated for a cylindrical cavern in Figure 2 below.

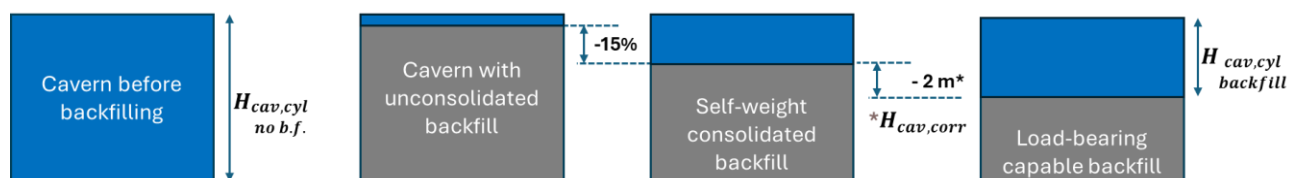


Figure 2 Relationship between actual cylindrical cavern height $H_{cav,cyl\ no\ backfill}$ and corrected cavern height after backfilling $H_{cav,cyl\ backfill}$



The thickness of the backfill body will be further reduced by consolidation under the load of collapsed and bulked overburden rock fragments after cavern migration is initiated. In [9] a value of 44% total backfill consolidation under the load of bulked overburden has been suggested based on several simple 1D settlement calculations using a NEN-Bjerrum parameter set derived from laboratory experiments on slurry taken directly from the brine purification plant.

In the reconstruction of cavern migration of backfilled caverns for the well-documented cavern migration cases, a value of 30% – 40% total consolidation was found to be more realistic for the range of migration heights observed in those cases [3], [4]. This value range has also been validated by subsequent testing of the BJP algorithm on several of the well-documented cases. Values of 15% SWC and 30% through loading by bulked overburden were found to best fit the subsidence prognosis to the actual (historically) observed migration and consequent subsidence measured at those cavern locations. The selected values compound to 40,5 % total consolidation.

The difference between the 44% consolidation value under the load of bulked overburden suggested by 1D calculations and the 40,5% total consolidation value for found by the reconstruction of well-documented migration cases (see Report 3 for details) can be attributed to the following:

1. The consolidation process of the fresh backfill in the cavern starts with self-weight consolidation (SWC, (15%) which takes places during and shortly after the backfilling process. This SWC thus precedes the consolidation by loading of the backfill with bulked overburden thereby reducing the latter as the SWC can be considered a “pre-consolidation” stage.
2. The loading of backfill by bulked overburden is a gradual process rather than the instantaneous load applied in the 1D settlement calculations.
3. The load of bulked overburden in the 1D settlement calculations is applied to the entire volume of backfill whereas in reality this is rarely the case. In most cases, the migration column surface area is (much) smaller than the cavern maximum lateral footprint and the load by bulked overburden is therefore applied to only a part of the total backfill volume.
4. The consideration of a 2 m thick layer of unconsolidated backfill material thus effectively increasing the total consolidation but on a cavern-by-cavern basis rather than a generally applied percentage.
5. The 1D settlement calculations cannot represent the actual 3-dimensional situation. Several factors can lead to a reduced load by bulked overburden due to for example friction between the bulked overburden and the migration column walls, “gap-bridging” and arching of bulked material in the migration column, and the presence of relatively competent sub-strata in the overburden which can cause “ledges” to form to which the bulked overburden can “anchor”.



6. "Curing" of the backfill by means of physical and chemical processes, e.g. crystallization of salt(s) due to oversaturation and the hydration of solid minerals such as Anhydrite and hemihydrate Gypsum.
7. The partly "undrained" conditions for consolidation of the backfill body due to the expected low permeability of the lower half of the backfill body (based on grainsize and crystallization) and the absence of lateral or downward paths, due to the presence of intact rock salt, for brine expelled from the backfill by loading of the backfill with bulked overburden.

For the BJP algorithm a value of 30 % consolidation of the backfill body by loading with bulked overburden has been selected based on the reconstruction of well documented cavern migration cases. The 30% consolidation under load of bulked overburden value is independent of how far upward a cavern actually migrates, considering its volume and geometry. In other words, the maximum expected consolidation of 30% under load of bulked overburden is applied to the backfill in all cases of migration as a conservative approach.

For the BJP algorithm, the input parameters for volume of unconsolidated backfill solids is denoted by V_{solids} and the input parameter for consolidation factor is denoted by f_{consol} .

With reference to Figure 1 please note the following:

In case 1 and 3 the height of the cylindrical part will be adapted based on backfill input parameters.

$$H_{cav,cyl} = H_{cav,cyl (no backfill)} + H_{cav,corr} - H_{cav,cyl,red}$$

$$H_{cav,cyl,red} = (1 - f_{consol}) \cdot \frac{V_{solids}}{\pi \cdot R_{cav,act}^2}$$

In case 2 the height of the conical part will be adapted based on backfill input parameters.

$$H_{cav,cone} = H_{cav,cone (no backfill)} + H_{cav,corr} - H_{cav,cone,red}$$

$$H_{cav,cone,red} = (1 - f_{consol}) \cdot \frac{3 \cdot V_{solids}}{\pi \cdot R_{cav,act}^2}$$

2.4 Simplified migration volumes

As the basic cavern shapes were simplified (as shown in Figure 1), the migration column volume can be simplified as well. The migration column volume of a cylindrical cavern will be cylindrical, for cone shaped caverns, the migration column volume will be a cone. The combination of a cylinder and cone will lead to a cylinder and cone-shaped migration column as shown in Figure 3 below.

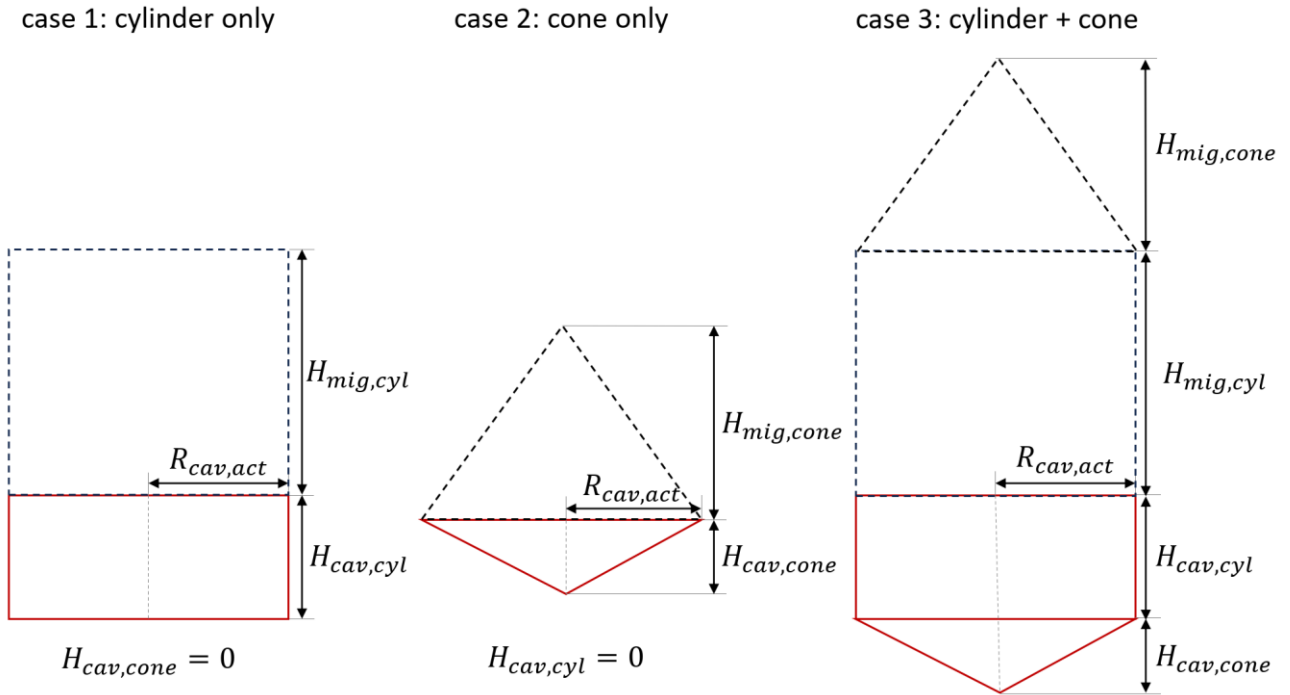


Figure 3 Basic cavern migration volumes with denotations

2.5 Migration of cylindrical-shaped caverns

For the migration height $H_{mig,cyl}$ of cylindrical caverns (or a cylindrical part of a combined geometry cavern) the bulking factor B is used as follows.

$$H_{mig,cyl} = \frac{H_{cav,cyl}}{B - 1}$$

$$d_{Tcav,cyl} = d_{Tcav,act} - H_{mig,cyl}$$

2.6 Radius reduction while migration of cone-shaped caverns

In case of cone-shaped caverns the inner angle of the migration cone volume β is derived from geometrical considerations (see Figure 4). The angle β is based on the inability of collapsed rock to (fully) bulk at the outer contour of a cavern with limited height during cavern migration, see [4] for a more detailed graphical explanation.

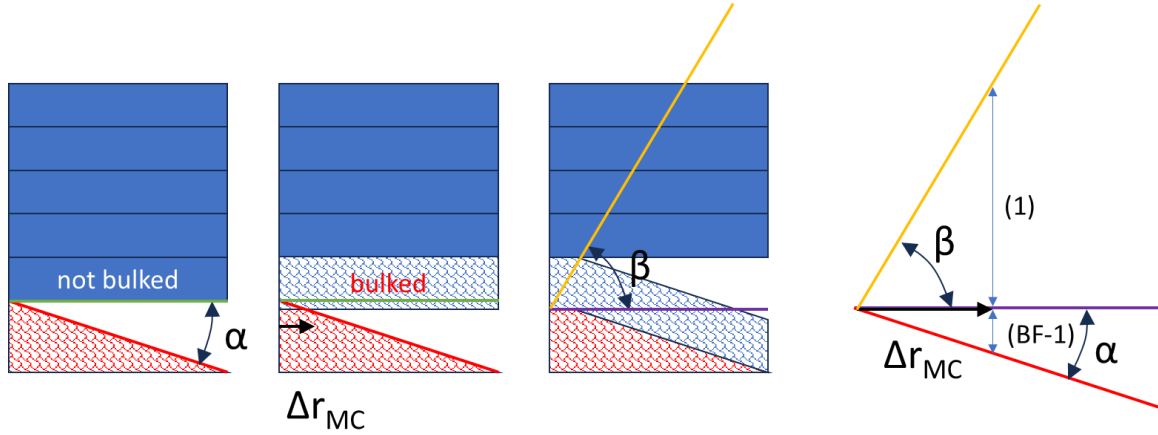


Figure 4 Geometrical solution for radius reduction in the case of cone-shaped caverns

The geometrical solution for calculation of angle β depends on the inner angle α and the bulking factor B .

$$\beta = \tan^{-1} \left(\frac{\tan \alpha}{B - 1} \right)$$

From geometrical setting (see case 2 in Figure 3) it is known, that $\tan \beta = H_{mig,cone} / R_{cav,act}$ and $\tan \alpha = H_{cav,cone} / R_{cav,act}$. By insertion into the equation above it follows, that

$$H_{mig,cone} = \frac{H_{cav,cone}}{B - 1}$$

$$d_{Tcav,final} = d_{Tcav,cyl} - H_{mig,cone}$$

Hence, the migration height for conical cavern volumes can be calculated similar to that of cylindrical caverns (compare to equation in section 2.5). The migration volume is limited by the volume of the original cavern and is indirectly proportional to the bulking factor.

$$V_{mig,cone} = \frac{V_{cav,cone}}{B - 1}$$



2.7 Critical depth at top rock (deflection of horizontal pillar)

The Mohr-Coulomb stress criterion describes a linear relation between minimum and maximum principal stresses.

$$\sigma_1 = \frac{(1 + \sin \varphi) \cdot \sigma_3 + 2 \cdot c \cdot \cos \varphi}{1 - \sin \varphi}$$

$\sigma_{1/3}$ – max./min. principal stress

φ – friction angle [°]

c – cohesion [MPa]

This relation is often used to estimate principal stresses as a first approximation. Empirical equations from technical mechanics can be used to estimate maximum principal stress under equally distributed vertical load q on a circular plate with radius R and height h [10].

$$\sigma_1 \cong \frac{1}{2} \left(\frac{R}{h} \right)^2 \cdot q$$

Combining both equations leads to an equation for the critical height h_{crit} of the circular plate.

$$h_{crit} = R \cdot \sqrt{\frac{1}{2} \cdot \frac{q \cdot (1 - \sin \varphi)}{(1 + \sin \varphi) \cdot \sigma_3 + 2 \cdot c \cdot \cos \varphi}}$$

For liquid-filled caverns, that are assumed to be in an open system, the vertical load can be assumed as effective stress, which is the difference between lithostatic pressure σ_0 and hydrostatic pressure p_{hyd} .

$$q = \sigma_0 - p_{hyd}$$

With an assumed minimum principal stress of $\sigma_3 = p_{hyd}$ the equation results in following term.

$$h_{crit} = R \cdot \sqrt{\frac{1}{2} \cdot \frac{(\sigma_0 - p_{hyd}) \cdot (1 - \sin \varphi)}{(1 + \sin \varphi) \cdot p_{hyd} + 2 \cdot c \cdot \cos \varphi}}$$

To get the initial (depth-dependent) pressures it is recommended to use d_{TR40} from the Bekendam calculation procedure as first approximation and iterate to increase the accuracy.

$$d_{crit} = d_{TR40} = d_{TR} + 40$$

$$\sigma_0 \approx 10^{-6} \cdot \gamma_s \cdot d_{crit} \quad [MPa]$$

$$p_{hyd} \approx 10^{-6} \cdot \gamma_w \cdot d_{crit} \quad [MPa]$$

$$R_{cav,crit} = R_{cav,act} - (d_{Tcav,cyl} - d_{crit}) \cdot \tan(90^\circ - \beta)$$



$$d_{crit} = d_{TR} + h_{crit} = d_{TR} + R_{cav,crit} \cdot \sqrt{\frac{10^{-6} \cdot (\gamma_s - \gamma_w) \cdot d_{crit} \cdot (1 - \sin \varphi)}{10^{-6} \cdot (1 + \sin \varphi) \cdot \gamma_w \cdot d_{crit} + 2 \cdot c \cdot \cos \varphi}}$$

The iteration process can be stopped after the third iteration. For friction angle φ and cohesion c it is recommended to use 21,3° and 2,8 MPa, respectively. These are typical conservative values for Röt claystone [7]. These values are also applied for the Muschelkalk formation overlying the Röt claystone as it is expected that it will behave similarly due to the bedded structure of this stratum. Referring to the abundant detailed descriptions in well logs, the Muschelkalk formation in the Hengelo area is a (thin bedded) calcareous clay/siltstone and in terms of geotechnical properties considered alike to the Röt claystone for this study. The Muschelkalk formation does comprise competent sub-strata that will have higher values for both friction angle φ and cohesion c . Considering the Muschelkalk alike to the Röt Claystone is thus a conservative approach.

3 Subsidence prognosis algorithm

3.1 Subsidence stages

Within the Update Migration Study TWR, potential surface subsidence, that can develop as a result of the presence of a cavern in the underground, is differentiated into five (5) possible stages. These stages are illustrated in Figure 5 below.

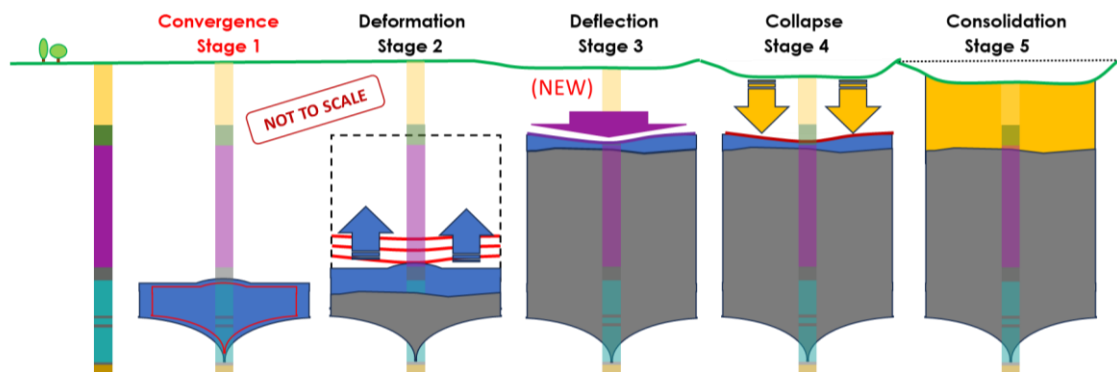


Figure 5 Possible subsidence stages for the studied caverns

The first stage of subsidence shown in Figure 5 is caused by cavern convergence due to salt creep. This type of subsidence is present for all caverns in the TWR area. Subsidence by convergence is not relevant for the Update Migration Study and therefore not further considered in the BJP-algorithm. Subsidence stages that are considered in the BJP-algorithm are all resulting from the migration of a cavern. Not all subsidence stages are always relevant, migration can stop due to cavern geometry well before all subsidence stages have developed. Short descriptions of each subsidence stage considered by the BJP-algorithm are provided by the following:

- Stage 2 Subsidence caused by the deformation of the cavern and migration column as it propagates upwards through the stratigraphy, i.e. mainly by deformation of the Röt Claystone and the overlying Muschelkalk formations.
- Stage 3 Subsidence caused by the deflection of the last remaining beam of competent rock over a migrated cavern below the unconsolidated Tertiary soils before the cavern migration reaches the critical depth d_{crit} .
- Stage 4 Subsidence caused by the collapse of the last remaining beam over competent rock and subsequent “plug-failure” of the Tertiary soils into the remaining cavern volume.
- Stage 5 Subsidence caused by the consolidation of the bulked overburden under the load of the Tertiary soils.

The implementation of each of these subsidence stages in the BJP-algorithm is presented in the following paragraphs.



3.2 Stage 2 subsidence

In contrast to Phase II subsidence in Bekendam calculation procedure, the stage 2 subsidence in the BJP algorithm is separated into a stage 2 – cylinder and a stage 2 – cone calculation. The cylindrical migration volume causes stage 2 subsidence alike the Bekendam calculation procedure (see Figure 10). The depth of the half-height of the migration body, used to calculate the Stage 2 subsidence, is therefore given by

$$d_{hmb,cyl} = (d_{Bcav,ini} + (d_{Tcav,act} - H_{hmb,cyl}))/2.$$

In cases, where the top of the cavern after cylindrical migration does not exceed the critical depth d_{crit} stage 2, it is essential that $H_{hmb,cyl} = H_{mig,cyl}$. In cases, where the top of the cavern after cylindrical migration exceeds the critical depth d_{crit} for stage 2, migration height is limited by $H_{hmb,cyl} = (d_{Bcav,ini} - d_{crit})/2$.

The effective cavern radius r'_{cyl} is given by

$$r'_{2,cyl} = \frac{\sqrt{\pi} \cdot R_{cav,act}}{2}$$

and the radius of influence by

$$R_{2,cyl} = d_{hmb,cyl} \cdot \tan \gamma.$$

Surface subsidence in the centre of the trough is derived by following equation, where erf denotes the gaussian error function.

$$\eta_{2,cyl} = S_{2,cyl} \cdot [erf(\sqrt{n \cdot \pi} \cdot r'_{2,cyl}/R_{2,cyl})]^2$$

$S_{2,cyl}$ is the effective cavern height for the cylindrical migration volume of stage 2. In practice subsidence is not determined by this formula, because $S_{2,cyl}$ is unknown. This formula is used to determine $S_{2,cyl}$.

$$S_{2,cyl} = \eta_{2,cyl} / [erf(\sqrt{n \cdot \pi} \cdot r'_{2,cyl}/R_{2,cyl})]^2$$

To determine subsidence in the centre of the trough, results of numerical simulations are used [5]. The results of these simulations are given in the table below and visualized in Figure 6.

Table 3 Results of numerical simulations of subsidence

Radius [m]	0	15	30	50	60
Subsidence [m]	0	-0,05	-0,1	-0,2	-0,3

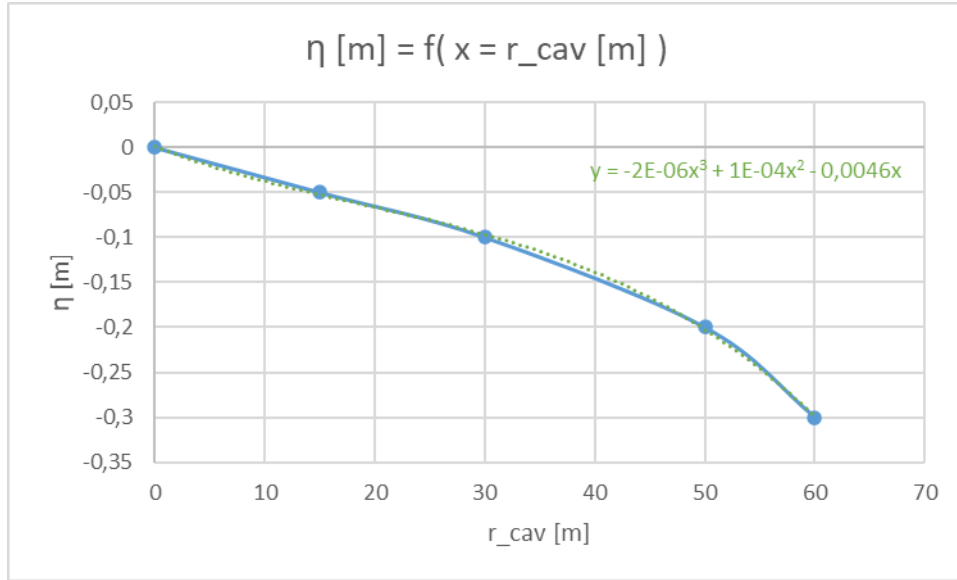


Figure 6 Stage 2 surface subsidence over cavern radius, data from [5]

Interpolation with a cubic equation can be used to calculate surface subsidence by input of cavern radius (green equation in Figure 6). These results are valid for a cavern with a height of 20 m. For different cavern heights, the surface subsidence must be adapted.

$$\eta_{2,cyl} = \eta_{2,cyl,Hcav=20m}(r = R_{cav,act}) \cdot H_{cav,cyl}/20$$

The conical migration volume causes stage 2 subsidence with reduced cavern radii during migration, see Figure 12. In the BJP algorithm, the half-height of the migration body used to calculate the subsidence due to the conical (upper part) of the migration column must be adapted as follows (see Figure 14).

$$d_{hmb,cone} = d_{Tcav,cyl} - (d_{Tcav,cyl} - d_{Tcav,hmb})/2$$

In cases, where the top of the cavern after conical migration does not exceed the critical depth $d_{Tcav,crit}$ for stage 2, it is essential that $d_{Tcav,hmb} = d_{Tcav,final}$. In cases, where the top of the cavern after conical migration exceeds the critical depth d_{crit} for stage 2, depth of the cavern top is limited by $d_{Tcav,hmb} = d_{crit}$ for this stage.

Due to the critical depth d_{crit} the maximum radius for stage 2 is limited as well, see also Figure 14 and Figure 15 for a graphical explanation.

$$r_{cav,hmb} = 0 \quad \text{for} \quad d_{Tcav,final} > d_{crit}$$

$$r_{cav,hmb} = R_{cav,crit} \quad \text{for} \quad d_{Tcav,final} \leq d_{crit}$$

The effective cavern radius $r'_{2,cone}$ for the conical part is calculated by the mean value of both radii.



$$r'_{2,cone} = \frac{\sqrt{\pi} \cdot r_{cav,mean}}{2} \quad \text{with} \quad r_{cav,mean} = \frac{R_{cav,act} + r_{cav,hmb}}{2}$$

The radius of influence for the conical part is calculated by

$$R_{2,cone} = d_{hmb,cone} \cdot \tan \gamma.$$

In line with stage 2 subsidence for cylindrical migration volume, the algorithm can also be used to calculate the conical migration volume.

$$S_{2,cone} = \eta_{2,cone} / [erf(\sqrt{n \cdot \pi} \cdot r'_{2,cone} / R_{2,cone})]^2$$

The surface subsidence must be corrected by the cavern height at the end of cylindrical migration, which is equal to $H_{cav,cone}$.

$$\eta_{2,cone} = \eta_{2,cone,Hcav=20m}(r = r_{cav,mean}) \cdot H_{cav,cone} / 20$$

Stage 2 subsidence in total is the superposition of cylindrical and conical subsidence.

$$\eta_2 = \eta_{2,cyl} + \eta_{2,cone}$$

3.3 Stage 3 subsidence

In the BJP algorithm stage 3 subsidence arises from the downward deflection of the cavern roof beam (horizontal pillar) as it approaches the critical depth due to migration. Stage 3 subsidence will be added to stage 2 subsidence, if the depth of the migrated cavern top is deeper than the critical depth.

$$d_{Tcav,final} > d_{crit}$$

When the cavern roof reaches the critical depth by migration, subsidence of stage 4 und 5 will replace subsidence of stage 3.

The vertical deflection of the top rock layer is denoted as S_3 and depends on cavern radius, effective stress at the critical depth and the distance to top rock h_{TR} as follows [11].

$$S_3 = 0.17 \cdot \frac{\sigma_0 - p_{hyd}}{E} \cdot \frac{R_{cav,final}^4}{h_{TR}^3}$$

$$\eta_3 = S_3 \cdot [erf(\sqrt{n \cdot \pi} \cdot r'_3 / R_3)]^2 \quad r'_3 = \frac{\sqrt{\pi} \cdot R_{cav,final}}{2} \quad R_3 = d_{Tcav,final} \cdot \tan \gamma$$

The elastic modulus E for Röt claystone was obtained from [8] with 5500 MPa. The claystone has an elasto-plastic behaviour, which leads to more deflection over time than a purely elastic material. To keep matters simple and conservative, the obtained elastic modulus was reduced by factor 10 for the BJP algorithm. The difference in the resulting subsidence between Bekendam and BJP algorithm is shown in Figure 7.

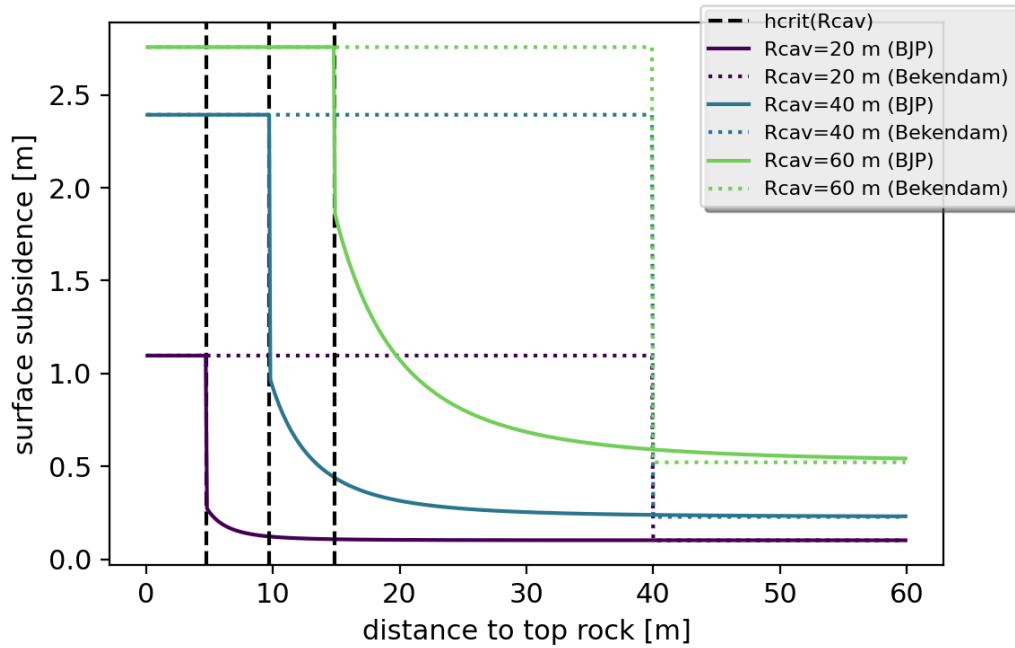


Figure 7 Subsidence calculation results for phase II/III (Bekendam) and stage 2+3/5 (BJP) for different cavern radii near critical depth

As can be seen from this figure, the introduction of stage 3 subsidence allows a more realistic, gradual transition between stage 2, i.e. subsidence caused by deformation of the total cavern and migration column partly filled with bulked overburden and stage 4, i.e. the formation of a sinkhole/trough due to the collapse of the last remaining competent cavern roof beam and subsequent plug failure of the Tertiary soils into the remaining cavern volume.

3.4 Stage 4 subsidence

Stage 4 subsidence corresponds to a collapse of the last remaining overburden rock, i.e. the horizontal roof pillar at the cavern top, and subsequent plug failure of the Tertiary soils. This combined process leads to either a sinkhole or a deep trough without the typical sharp edges of a sinkhole. Therefore stage 4 subsidence will occur if the depth of the final cavern top is above the depth of the top rock.

$$d_{Tcav,final} \leq d_{TR}$$



If the cavern height is higher than 2 m at the depth of the top rock, a sinkhole will appear. Otherwise, stage 4 subsidence will create a trough [6].

$$H_{cav,TR} \geq 0 \text{ m} \rightarrow \text{trough}$$

$$H_{cav,TR} \geq 2 \text{ m} \rightarrow \text{sinkhole}$$

The cavern height at critical depth $H_{cav,crit}$ determines if a trough or a sinkhole appears as well, because it is related to the cavern height at top rock. In cases, where $d_{Tcav,cyl} \leq d_{crit}$ (or $d_{Tcav,cyl} \leq d_{TR}$) cavern heights at critical depth and top rock are determined as follows.

$$H_{cav,crit} = H_{cav,crit,cyl} + H_{cav,cone}$$

$$H_{cav,TR} = H_{cav,TR,cyl} + H_{cav,cone}$$

$$H_{cav,crit,cyl} = H_{cav,cyl} - (d_{Tcav,act} - d_{crit}) \cdot (B - 1)$$

$$H_{cav,TR,cyl} = H_{cav,cyl} - (d_{Tcav,act} - d_{TR}) \cdot (B - 1)$$

See Figure 10, Figure 12 and Figure 14 for a graphical explanation of these parameters.

To determine $H_{cav,crit}$ and $H_{cav,TR}$ if cylindrical migration is finished below critical depth (or depth of top rock) $d_{Tcav,cyl} > d_{crit}$ (or $d_{Tcav,cyl} > d_{TR}$) the bulking factor can be used as follows.

$$H_{cav,crit} = (d_{crit} - d_{Tcav,final}) \cdot (B - 1)$$

$$H_{cav,TR} = (d_{crit} - d_{Tcav,final}) \cdot (B - 1)$$

$$R_{cav,crit} = R_{cav,act} - (d_{Tcav,cyl} - d_{crit}) \cdot \tan(90^\circ - \beta)$$

$$R_{cav,TR} = R_{cav,act} - (d_{Tcav,cyl} - d_{TR}) \cdot \tan(90^\circ - \beta)$$

The effective cavern height in stage 4 is equal to the height of the cavern at top of overburden rock.

$$S_4 = H_{cav,TR}$$

The radius of influence in case of stage 4 subsidence is much smaller, than in case of stage 2 subsidence, see also Figure 11, Figure 13 and Figure 15 for a graphical explanation. Therefore, it is calculated based on depth for top of overburden rock. The effective radius must be adapted as well in stage 4.

$$r'_4 = \frac{\sqrt{\pi} \cdot R_{cav,TR}}{2} \quad R_4 = d_{TR} \cdot \tan \gamma$$

In the case of a sinkhole, the formula must be slightly corrected by excluding the error function for η_4 .

$$\eta_{4,sinkhole} = S_4 = H_{cav,TR}$$

To avoid a sudden transition (“saltus”) between a calculated trough and sinkhole, the radius of influence for stage 4 is linear increasing (depending on cavern height at top rock) in BJP algorithm (see Figure 8).

$$R_{4,linear} = \frac{R_{cav,TR} - R_4}{2} \cdot H_{cav,TR} + R_4$$

This adaption will lead to a (more realistic) non-linear smooth transition of stage 4 subsidence for the trough scenarios (compare to equation for stage 2 subsidence, see Figure 9).

$$\eta_{4,trough} = S_4 \cdot [erf(\sqrt{n \cdot \pi} \cdot r'_4 / R_{4,linear})]^2$$

Note: Stage 4 subsidence is calculated based on phase IIIA subsidence equations by Bekendam. In the BJP algorithm the dimensions (cavern radius, cavern height, trough radius) are however determined differently.

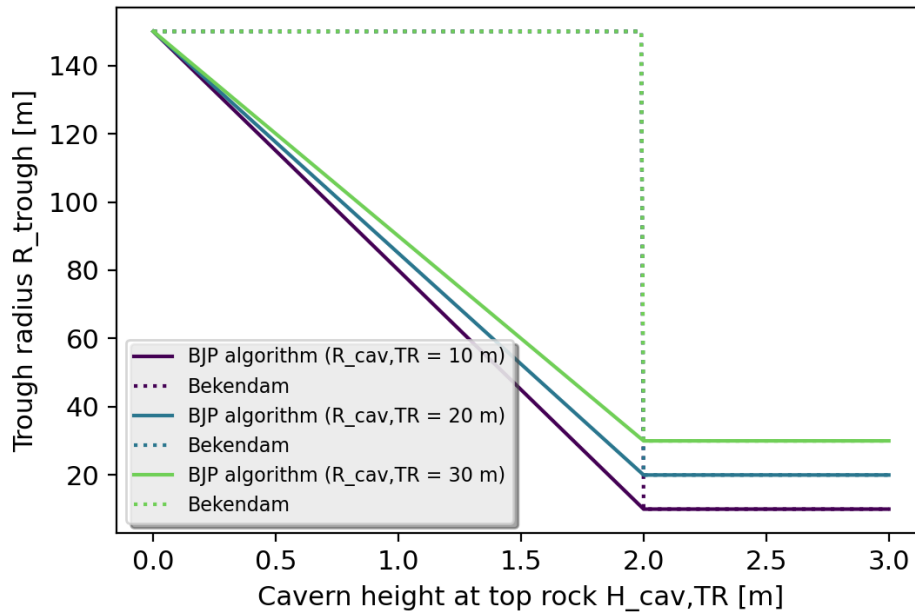


Figure 8 Linear transition of trough radius with increasing cavern height at top rock for different cavern radii compared to Bekendam

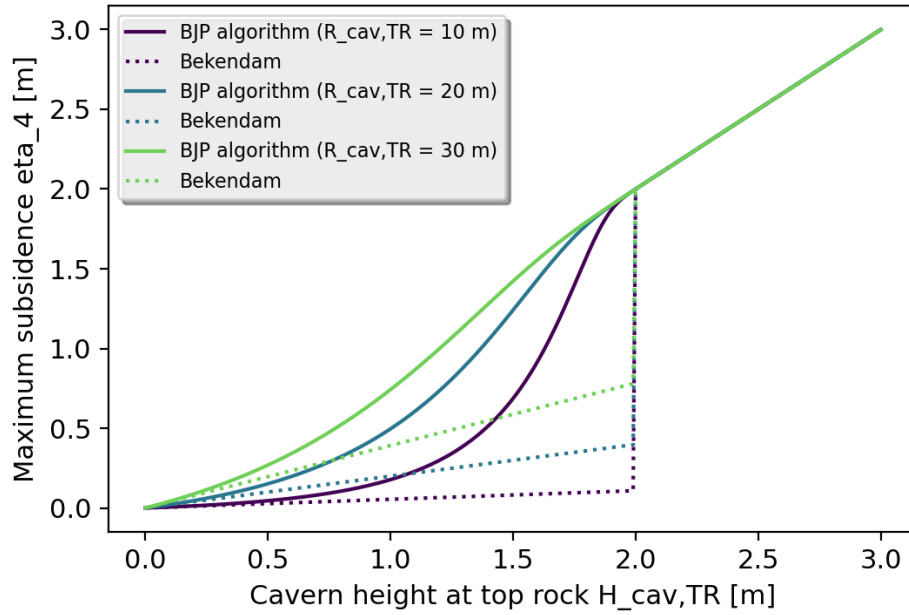


Figure 9 Smooth non-linear transition of stage 4 subsidence with increasing cavern height at top rock for different cavern radii compared to Bekendam

3.5 Stage 5 subsidence

Since stage 3 considers subsidence of caverns migrating just below the critical depth, stage 5 subsidence is always applied on caverns migrating above the critical depth.

$$d_{Tcav,final} \leq d_{crit}$$

Stage 5 subsidence is caused by consolidation the bulked overburden under the load of the Tertiary soils. For consolidation coefficient C_m the maximum value (see Table 2) is used as a conservative approach.

$$S_5 = C_m \cdot \gamma_s \cdot d_{TR} \cdot (d_{Bcav,ini} - d_{TR} - H_{cav,TR})$$

$$\eta_5 = S_5 \cdot [erf(\sqrt{n \cdot \pi} \cdot r'_4 / R_4)]^2$$

Note: Stage 5 subsidence is calculated similar to the phase IIIB subsidence by Bekendam, but with an adapted cavern height and adapted radii as input from BJP algorithm. Another difference is, that the criterion for stage 5 in BJP algorithm is the cavern radius dependent critical depth d_{crit} and not d_{TR40} (which was used by Bekendam independently of cavern radius).

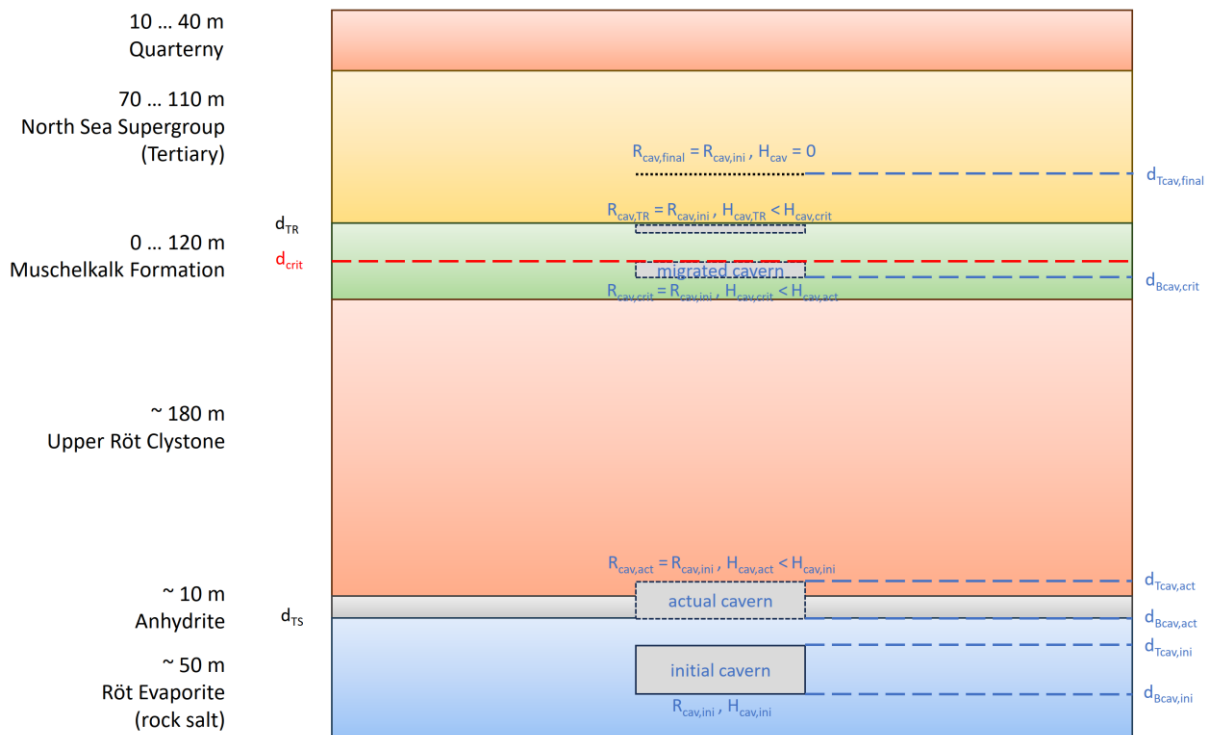


Figure 10 Scheme of a migrating cavern with denotations of depths and heights for case 1

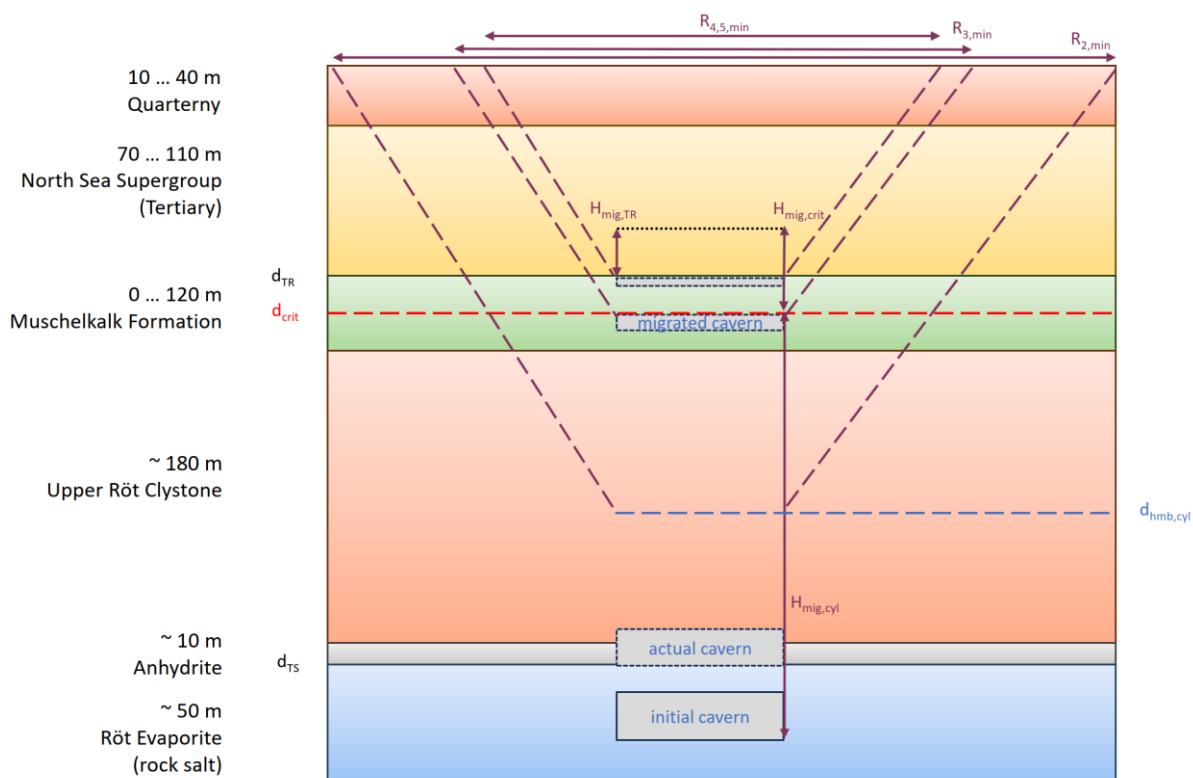


Figure 11 Scheme of a migrating cavern with denotations of migration column and radius of influence for case 1

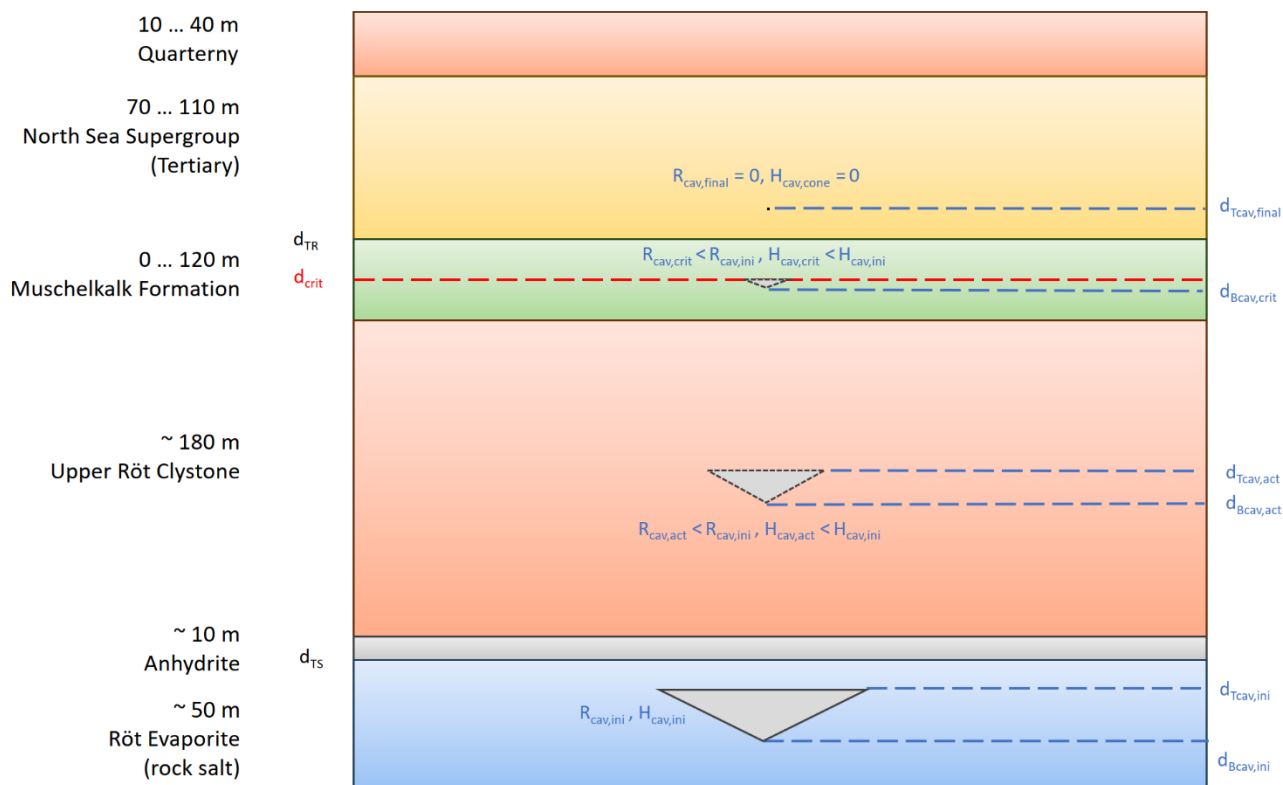


Figure 12 Scheme of a migrating cavern with denotations of depths and heights for case 2

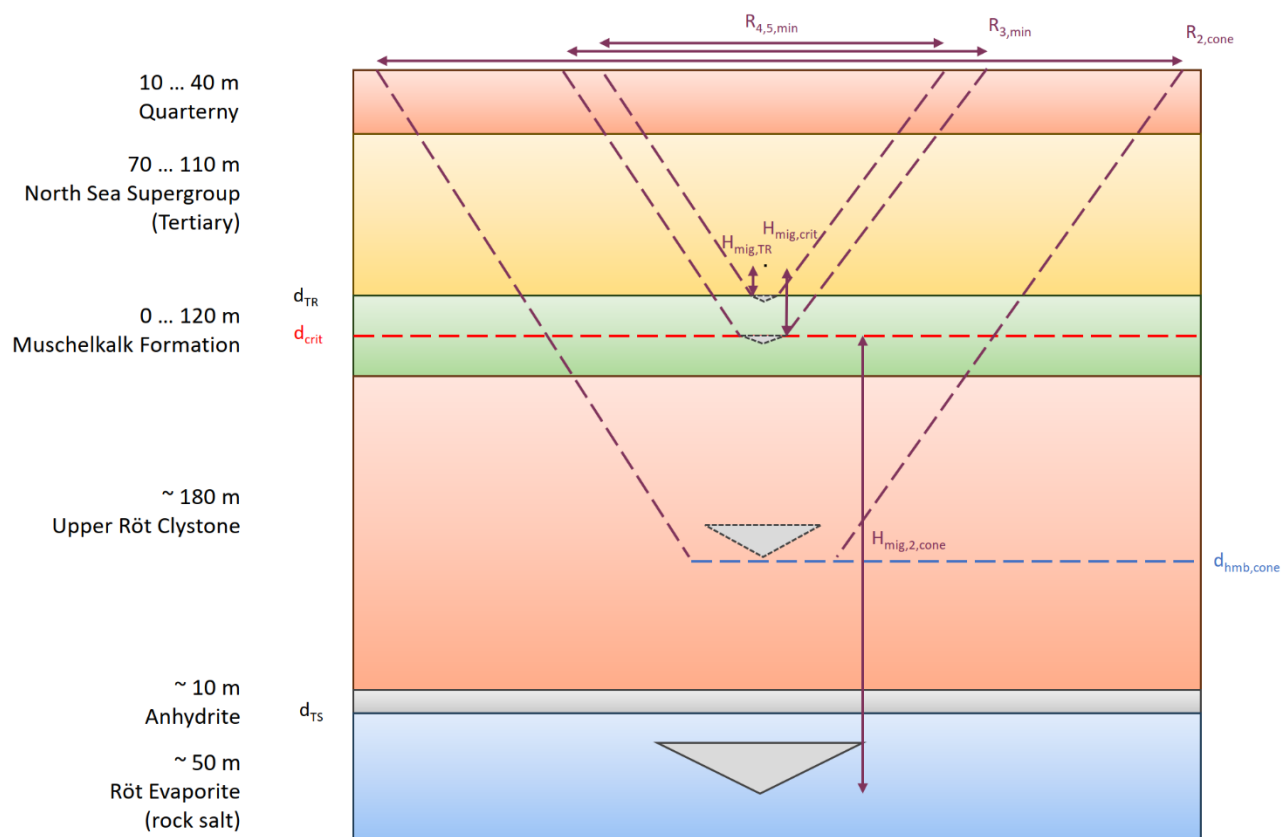


Figure 13 Scheme of a migrating cavern with denotations of migration column and radius of influence for case 2

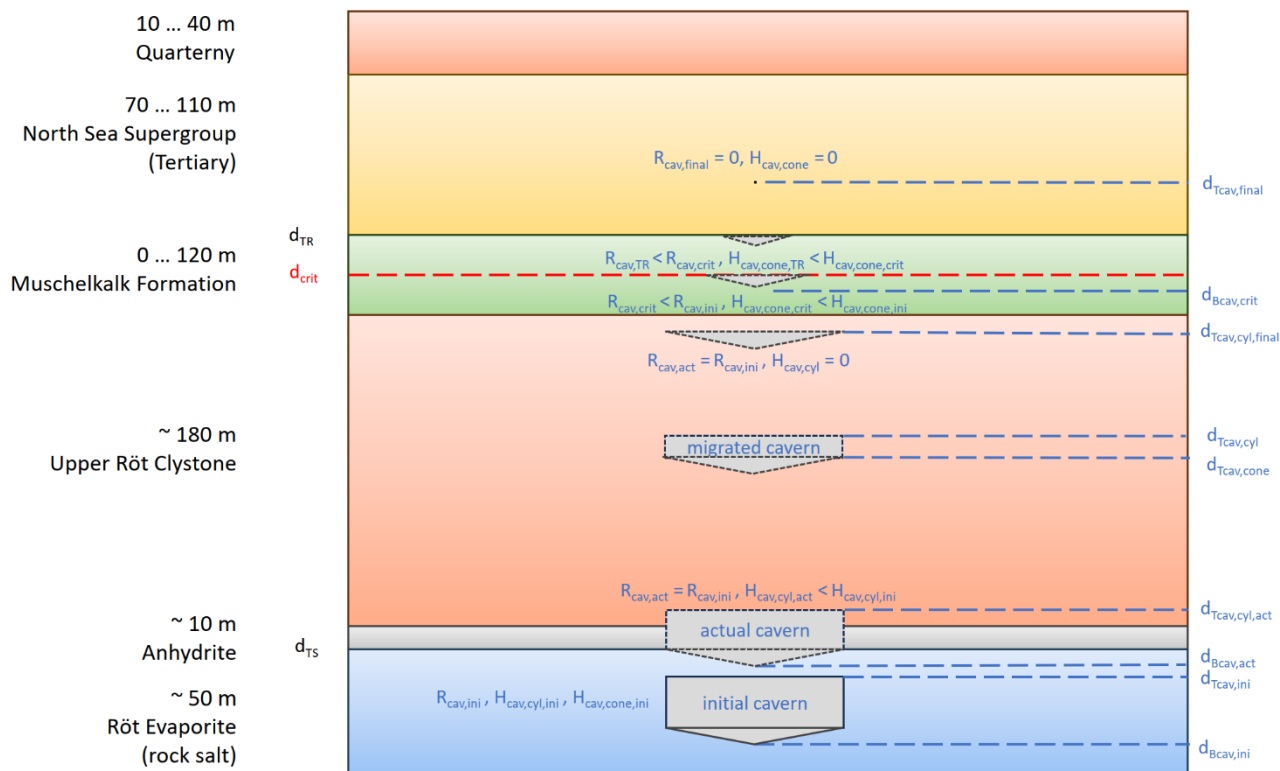


Figure 14 Scheme of a migrating cavern with denotations of depths and heights for case 3

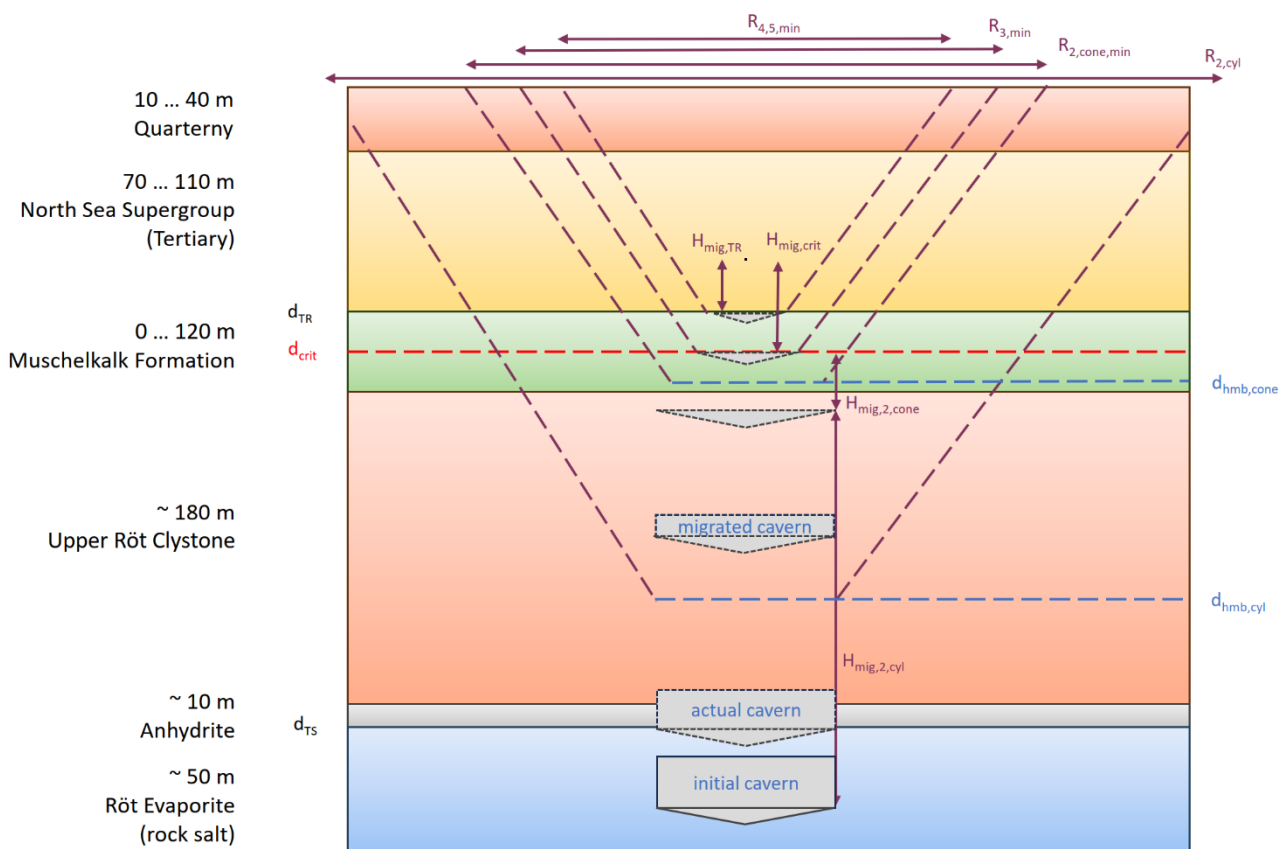


Figure 15 Scheme of a migrating cavern with denotations of migration column and radius of influence for case 3

3.6 Superposition of subsidence values of stage 2 to 5

Subsidence values from single stages 2 to 5 are summed up to get the subsidence for stage 2+3, stage 4+5 or the total subsidence as follows.

$$\eta_{23} = \eta_{2,cyl} + \eta_{2,cone} + \eta_3 = \eta_2 + \eta_3$$

$$\eta_{45} = \eta_4 + \eta_5$$

$$\eta_{total} = \eta_{23} + \eta_{45}$$

3.7 Radial expression of subsidence, tilt, strain, curvature and relative rotation

For radial expression of **subsidence** and **tilt** after Bekendam method, the following equations are used.

$$\eta(r) = \frac{S}{2} \cdot \operatorname{erf}\left(\frac{\sqrt{n \cdot \pi} \cdot r'}{R}\right) \cdot \left[\operatorname{erf}\left(\frac{\sqrt{n \cdot \pi} \cdot (r' + r)}{R}\right) + \operatorname{erf}\left(\frac{\sqrt{n \cdot \pi} \cdot (r' - r)}{R}\right) \right]$$

$$\zeta(r) = -S \cdot \frac{\sqrt{n}}{R} \cdot \operatorname{erf}\left(\frac{\sqrt{n \cdot \pi} \cdot r'}{R}\right) \cdot \left(e^{-\frac{n \cdot \pi \cdot (r' + r)^2}{R^2}} - e^{-\frac{n \cdot \pi \cdot (r' - r)^2}{R^2}} \right)$$

When the unit of S is mm, then the unit of the equations output for subsidence is mm and the output for tilt is mm/m.

For radial expression of **curvature** after Bekendam method, the following equation is used.

$$\kappa(r) = -S \cdot \frac{n \cdot \sqrt{n} \cdot \pi}{R^3} \cdot 2 \cdot \operatorname{erf}\left(\frac{\sqrt{n \cdot \pi} \cdot r'}{R}\right) \cdot \left[(r' + r) \cdot e^{-\frac{n \cdot \pi \cdot (r' + r)^2}{R^2}} + (r' - r) \cdot e^{-\frac{n \cdot \pi \cdot (r' - r)^2}{R^2}} \right]$$

When the unit of S is mm, then the unit of the equations output for curvature is $10^{-3}/m$.

For radial expression of **strain** after Bekendam method, curvature is multiplied by radius of influence R and the strain coefficient C_s .

$$\varepsilon(r) = C_s \cdot R \cdot \kappa(r)$$

When the unit of S is mm, then the unit of the equations output for strain is mm/m.

The choice of S , r' and R for the equations for subsidence, tilt, strain and curvature depends on the case. All possibilities are listed in the following table.



Table 4 Input setting for radial expression of subsidence, tilt, strain and curvature

Stage	criteria	$S =$	$r' =$	$R =$
Stage 2 cyl	$H_{cav,cyl} > 0 \text{ m}$	$S_{2,cyl}$	$r'_{2,cyl}$	$R_{2,cyl}$
Stage 2 cone	$H_{cav,cone} > 0 \text{ m}$	$S_{2,cone}$	$r'_{2,cone}$	$R_{2,cone}$
Stage 3	$d_{Tcav,final} > d_{crit}$	S_3	r'_3	R_3
Stage 4 trough	$H_{cav,TR} \geq 0 \text{ m}$	S_4	r'_4	R_4
Stage 4 sinkhole	$H_{cav,TR} \geq 2 \text{ m}$	$\eta_4 = S_4 = H_{cav,TR}$	-	-
Stage 5	$d_{Tcav,final} \leq d_{crit}$	S_5	r'_4	R_4

The subsidence/tilt/strain/curvature for stages 2-5 in total is the superposition of the single stages. For radial expression of **relative rotation**, curvature and a building length $L=50 \text{ m}$ is used in the equation below.

$$\xi(r) = 180^\circ \cdot (\sin^{-1}(0.5 \cdot L \cdot \kappa(r))) / \pi$$

When the unit of κ is $1/\text{m}$, then the unit of the equations output for relative rotation is $^\circ$.

The superposition of several relative rotation values is done by summation of the curvature values before using the equation above.

3.8 Calculation of the amount of required backfill for only stage 2 subsidence

The volume of effective backfill is calculated in the same way as $V_{cav,crit}$ (see stage 4 subsidence) but with a more conservative critical height $h_{crit,safe}$ which is defined as follows.

$$h_{crit,safe} = f_{safe} \cdot h_{crit}$$

f_{safe} is the safety factor for the critical height and an input parameter for the BJP algorithm. The used f_{safe} equals two (2) and prevents the formation of stage 3 subsidence as a conservative approach.

The minimal volume of backfill required to avoid the formation of a sinkhole or trough (with or without applying a safety factor f_{safe}) is calculated by the BJP-algorithm. The minimal volume of backfill needed to avoid only stage 4 and 5 subsidence, thus not applying f_{safe} is also calculated by the algorithm as a comparative value. The resulting backfill volumes need to be post-processed considering self-weight consolidation (15%) and consolidation under the load of bulked overburden (30%) to enable the estimation of total “fresh” backfill needed.



3.9 Additional notes and validation of the BJP-Algorithm

The BJP subsidence prognosis algorithm does not calculate long term subsidence processes. This includes stage 1 subsidence, which relates to slow creep processes in the salt body, as well as subsidence caused by very slow deformation processes within the claystone after stage 2 and stage 3 subsidence has occurred. These processes are under stable conditions and uncritical for the surface from a geomechanical point of view, because

1. they will happen in a very long time period of hundreds and thousands of years, and
2. their radius of influence is large and hence the maximum amount of subsidence resulting from these processes is very low.

When a cavern reaches critical depth, stage 5 is calculated and stage 3 is not calculated by the BJP algorithm, because of the conservative approach, that it will be completely replaced by stage 5 (consolidation of the bulked overburden material by loading with Tertiary soil).

In a special case, when the final depth of a cavern with a cone (case 2 or 3) is between critical depth and the depth of top rock, BJP algorithm denotes its situation as “stage 5 only”. But in this case the subsidence arising from stage 5 is zero, because $r'_4 = r'_3 = 0$ (with $R_{cav,final} = 0$). Practically in this situation the upper part of the Röt Claystone/Muschelkalk is stabilized by the arching effect and only stage 2+3 will affect the surface subsidence. Stage 3 will be neglectable small, because of the small remaining cavern radius at critical depth. In this special case only stage 2 will have an impact on the result, although it is stated as stage 5 in the algorithm.

A detailed description of the reconstructions, and the consequent validation of the algorithm, will be included in Report 3, titled “Validation of the input parameters by the assessment of migration behaviour of migrated caverns and determination of the likelihood of migration”. Conclusions on the applicability and conservatism of the newly developed BJP-Algorithm and a comparison with the formerly used Bekendam method will be included in Report 3 rather than in this report to aid legibility by the presentation of several well-documented examples of cavern migration.



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